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Some Fundamental Concepts of Femtosecond Laser Filamentation

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The fundamental physical concepts underlying filamentation are discussed. These include slice-by-slice self-focusing, intensity clamping, back ground reservoir and self-compression of the pulse during propagation.

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I. INTRODUCTION

This paper attempts to clarify some fundamental concepts of femtosecond (fs) laser filamentation in an optical (transparent) medium since there are currently many new applications which are being explored. These range from remote sensing in the atmosphere to few cycle pulse generation, micro-material processing, writing wave guides, fiber Bragg gratings, *etc.* in the laboratory. For a most recent comprehensive review, see Ref. 1.

First of all, we should emphasize that the fundamental physics of filamentation is the same in all optical media, be they gases, liquids, or solids. It is simply the balancing actions between Kerr self-focusing of the pulse in a neutral medium and the self-de-focusing by the self-generated weak plasma (free electrons). The difference lies in the details of free electrons generation. In gases, tunnel ionization of the gas molecules inside the self-focal volume results in a plasma that defocuses the pulse [2]. In condensed matters, excitation of free electrons from the valence to the conduction bands [3] is followed by inverse Bremstrahlung and electron impact ionization [4] before the short pulse is over. The well-known type of optical breakdown of the medium (generation of a spark) by longer laser pulses in the picosecond (ps) and nanosecond (ns) regimes does not occur in the fs self-focusing regime because there is not enough time to sustain cascade (avalanche) ionization. For example, at a one-atmosphere pressure, the mean free time of an electron collision is ∼1 ps. This time is longer than the fs pulse duration so that only tunnel ionization, an 'instantaneous' electronic transition process, is responsible for the generation of free electrons [5] even though the full pulse is involved in the self-focusing including external focusing [6]. In the case of a condensed medium, external focusing may influence the outcome significantly [1]. If we consider only those cases in which self-focusing is a dominant focusing process (with a long focal length lens or without external focusing), the filament will be formed without breakdown. The reason for this is that self-focusing induces a strongly convergent wave front only in a narrow transverse region that contains a peak power on the order of the critical power for self-focusing in the material [7]. The few electrons produced due to multiphoton/tunnel transitions from the valence to the conduction bands followed by at most a few cycles of collisional ionization will be enough to limit the intensity increase and prevent further ionization.

II. SLICE-BY-SLICE SELF-FOCUSING

We now consider a short laser pulse, say, 100 fs in duration, from the most popular Ti-sapphire laser system (central wavelength 800 nm). It propagates in air (as an example) without the aid of any focusing element; *i.e.*, free propagation.

For self-focusing to occur, the transverse spatial intensity distribution of the pulse across the wave front should not be uniform. We approximate the pulse as a plane-wave pulse and assume that the intensity distribution across the pulse's transverse cross section is Gaussian. We shall follow the propagation of the central (most powerful) 'slice' of the pulse. The thickness of this 'slice' is at least $c\tau$, where c is the speed of light in vacuum and τ is the period of oscillation of the electromagnetic wave. This is because we are talking about an intensity that is defined as the Poynting vector averaged over at least one cycle of oscillation. The propagation of this slice is similar to that of a wave front. If the intensity at the

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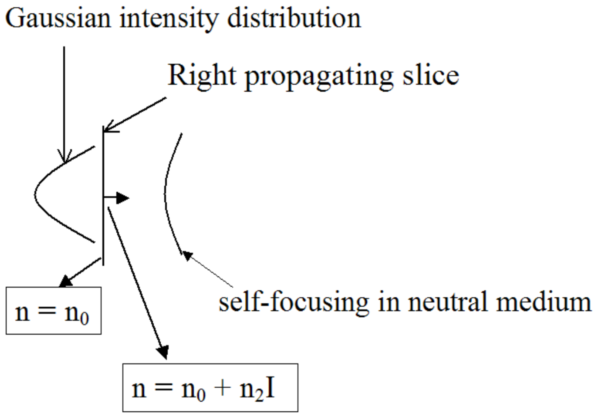

Fig. 1. Schematic illustration of the self-focusing of a slice of a femtosecond laser pulse in an optical medium. The initial plane-wave slice has a Gaussian intensity distribution across the transverse plane. The central part, having a higher index of refraction due to Kerr nonlinear index increase, propagates slower than the rest of the slice, resulting in a concave slice front, which means focusing.

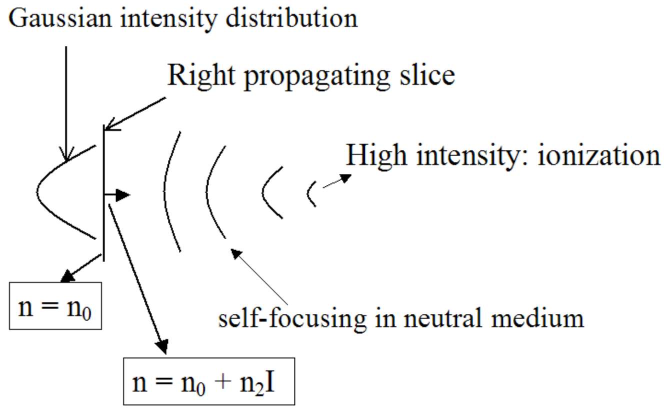

Fig. 2. When the self-focusing effect is stronger than linear diffraction and the effect of GVD, the radius of curvature of the slice will keep decreasing; hence, self-focusing becomes stronger and stronger. Soon, the intensity in the self-focusing zone becomes so strong that tunnel ionization starts to be significant. The resultant plasma slows down the focusing and balances it at the self-focal plane, where a maximum intensity is reached (intensity clamping) before the slice diverges out into the background reservoir.

central zone of the slice is high enough so that the nonlinear Kerr effect cannot be neglected, the index of refraction of the central zone will be given by $n = n_0 + n_2 I$ while the index at the edge of the slice will be $n = n_0$. Here, n_0 is the linear index of refraction in air and is $n_2 I$ the Kerr nonlinear index of refraction; n_2 and I being the coefficient of the Kerr nonlinear index of refraction and the local intensity, respectively. The speed of propagation of the slice is given by c/n. Hence, the central part of the slice propagates slower than the rest of the slice, giving rise to a concave wave front as shown in Fig. 1. This is the beginning of self-focusing. However, this self-focusing effect is not sufficient to guarantee filamentation because there is always a linear diffraction of the pulse that will cause the pulse to diverge as it propagates further. If the self-focusing effect is not strong enough to counteract the diffraction effect, the consequence is a slowly divergent pulse, slower than that due to pure linear diffraction. Consequently, the pulse's diameter looks almost constant over some distance of propagation.

When the natural linear diffraction of the pulse is just balanced by self-focusing, the peak power equals the so-called critical power for self-focusing. Through a solution of Maxwell's equations for a non-paraxial CW Gaussian beam, the critical power for self-focusing is given by $P_c = \frac{3.77\lambda^2}{8\pi n_2 n_0}$ where λ is the central wavelength of the pulse [7]. This expression shows that the critical power for self-focusing depends only on n_2, n_0 and λ and is independent of the intensity. Thus, when the peak power of the pulse is higher than the critical power for self-focusing, the slice shown in Fig. 1 will continue on curving forward as the wave front propagates further. If the peak power is only very slightly higher than P_c, the group velocity dispersion (GVD) will lengthen the pulse after a short distance of propagation; this lowers the peak power to a value below P_c and the pulse will again diverge slowly through diffraction. However, with femtosecond laser pulses, it is easy to obtain a high peak power that can readily overcome both linear diffraction and GVD. A few tens of percent higher than P_c is enough [3]. Once such self-focusing starts, it will not stop. Thus, the slice keeps curving into a smaller and smaller zone as it propagates while the intensity becomes higher and higher (Fig. 2). Soon, the high intensity in the self-focal zone will tunnel ionize [5] air molecules, resulting in the generation of a weak plasma. The change in the index of refraction of the slice propagating in a plasma is $(\Delta n)_p \cong -\frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$, where N_e is the electron density, e and m_e are the electronic charge and mass, respectively, and ω_0 is the central frequency of the pulse. The index of refraction of the central part of the slice is, thus, $n = n_0 + n_2 I - \frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$. This will increase the speed of propagation of the central part of the slice; *i.e.*, the curvature of the slice starts to flatten out, but it is still focusing so long as $n_2 I > \frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$. Thus, the intensity is still increasing. The electron density increases very rapidly with the intensity because tunnel ionization is a highly nonlinear process. We approximate such an increase as being governed by an effective power law according to an experimental observation [8]; *i.e.*, $N_e(t) \propto I^m$, where m is the effective nonlinear order of ionization. In air, m is about 8 [8]. The effective index of refraction of the central part of the slice is, thus, $n = n_0 + n_2 I - \frac{4\pi e^2}{2m_e\omega_0^2} k I^m$, where k is a proportionality constant. Qualitatively, this means that the free electron term will quickly catch up with the Kerr term until that they are equal; *i.e.* until $n_2 I = \frac{4\pi e^2}{2m_e\omega_0^2} k I^m$. At this point, Kerr self-focusing balances free electron defocusing and the central part, having now an index of refraction n_0 propagates at the same speed as the rest of the slice. There is no more focusing and the intensity is highest at this balancing point. This is the condition of intensity clamping [9–11] because further propagation would lead to an index at the central part smaller than n_0. The slice will start to diverge. That is to say, during self-focusing of a powerful femtosecond laser pulse in an optical medium, there is a maximum intensity that self-focusing can reach. In air, it is around 5×10^{13} W/cm^2 [10]. The energy in the de-focusing slice will be reduced a little due to the loss in ionization. After passing through the self-focus, the central slice is returned (defocused) back to the remaining part of the whole pulse or to the background reservoir [12,13]. This background reservoir is an important concept in considering the physics of filamentation. A most recent experimental and numerical study of the background reservoir is given in Ref. 14.

The central slice will self-focus at a distance z_f from the beginning of the propagation in the medium given by [7]

$$z_f = \frac{0.367 k a_0^2}{\left\{\left[\left(\frac{P}{P_c}\right)^{1/2} - 0.852\right]^2 - 0.0219\right\}^{1/2}} \qquad (1)$$

where k and a_0 are the wave number and the radius, respectively, of the beam profile at 1/e level of intensity, and P is the peak power of the slice. The slice in front of the central slice will then self-focus at a later position in the propagation direction according to Eq. (1) because its peak power is lower than that of the central slice. It will undergo the same processes, namely, self-focusing, intensity clamping, and de-focusing, and will return the (slightly lowered) energy back to the background reservoir, and so on for successive front slices whose peak powers are higher than the critical power. Thus, the front part of the pulse will become thinner and thinner as the pulse propagates. The back slices symmetrical to the front slices will in principle also self-focus at a position slightly behind the self-foci of the front slices. However, this will never happen because it will encounter the plasma left behind by the central and successive front slices. These back slices will thus self-focus into and interact with the plasma giving rise to a complex intensity distribution [see for example, Ref. 4 and 15]. In general, the energy in the back part of the pulse will still be confined inside the highly deformed body of the pulse or the background reservoir. During the propagation, repeated processes of Kerr self-focusing in the neutral gas and self-defocusing in the self-generated weak plasma of the slices in the front part of the pulse result in a continuous series of hot spots along the propagation axis. This gives rise to the perception of a filament, hence, filamentation [2, 12,16,17]. Since the energy loss in the ionization process is small, the pulse can repeat the whole process again, resulting in what we call self re-focusing [12,18].

III. COMMENTS

1. Concept of a Filament

The word 'filamentation' may sometimes be misleading although we keep on using this word. We emphasize that the laser pulse does not degenerate or self-stretch into a thin and long line of intense light (filament) in the case of a single filamentation. It is not the propagation of the self-focus along the axis of propagation that gives rise to the perception of a filament. Each self-focus comes from the self-focusing of a different slice of the pulse. A filament is just the perception of a succession of moving self-foci surrounded by a low-intensity background reservoir. Most of the energy of the pulse is inside the reservoir. The succession of self-foci leaves behind a weak and narrow plasma column along the path of the strong self-foci. The plasma generated inside the self-foci gives rise to a long-lived nitrogen fluorescence (compared to the transit time of the laser pulse) in air, which appears as a continuous line. At any time, there is only one pulse propagating in space. Inside this pulse, there is only one most intense hot spot (self-focus) at the front part of the pulse even though pulse splitting during propagation may give rise to a secondary less intense hot spot behind it at some positions. This most intense hot spot changes its position inside and towards the front of the pulse during propagation.

2. Self-compression

We mentioned above that the front part of the pulse would become thinner and thinner as the pulse propagates. This is what could be interpreted as pulse self-compression [19]. The thinner front part of the pulse evolves continuously as the pulse propagates. If somehow, one could extract this thin part of the pulse and eliminate the rest of the pulse, the consequence is clean few cycle pulses. A current experimental challenge is to efficiently and very simply generate few-cycle down to single cycle pulses.

3. P_c

The critical power for self-focusing, which is inversely proportional to the coefficient of the nonlinear index of refraction, n_2, is not a constant in air. It depends on the response of the medium to the pulse duration. We note that the response of a medium to an electromagnetic wave is essentially the induced polarization (dipole moment per unit volume). When the pulse duration is shorter than 100 fs, only an 'instantaneous' electronic response (induced polarization due to a pure electronic oscillation that can follow the field) is fast enough to contribute to the total (linear and nonlinear) polarization which, in turn, contributes to the total index of refraction and to n_2. When the pulse is longer so that the interaction time is longer, both the electronic and he nuclear responses involving the Raman transition (excitation of a molecular vibration) contribute to a larger value of n_2. This lowers the critical power [20]. In air, our recent experimental measurement shows that P_c changes from about 10 GW for pulse durations shorter than 100 fs to about 3 GW for pulses longer than 100 fs [20]. We are in the process of making similar measurements for some condensed media.

4. Non-ionizing Filamentation

Recently, some papers have claimed to observe filamentation without ionization and attribute this to soliton propagation [21,22]. It might well be that, at the end of the propagation, the peak power is slightly lower than the critical power. As discussed above, diffraction will overcome self-focusing, and the beam will diverge very slowly over a long distance as if the diameter is hardly changed. This leads to the perception of self-guiding, but indeed, it is simply the diffraction that is slowed by self-focusing.

5. Background Reservoir

The back ground reservoir is a low intensity zone, yet it contains most of the energy in the pulse [14]. This physical concept can explain re-focusing. It can also explain why filamentation still persists for propagation through raindrops [23]. This is because while a raindrop might have 'killed' the filament core, the background reservoir can still self-focus further into a filament.

6. Intensity Clamping

Intensity clamping is a profound physical phenomenon of self-focusing and filamentation. It sets an upper limit to the intensity at the self-focus not only in air but also in all optical media. Even if one tries to focus the pulse, so long as the focal length is not too short [6], self-focusing will always start before the geometrical focus. Thus, the intensity at the geometrical focus is either lower than or as high as that inside the self-focal zone in air. The consequence of this intensity clamping is far reaching. In air, one can have self-focusing at a long distance but one can not further increase the intensity inside the self-focus, not even by significantly increasing the energy of the pulse to many times the critical power; in practice, there will only be an increase in the number of self-foci (multiple filamentation), each of which will have a similar peak intensity. The 'dream' of reaching an enormous intensity (that might induce a nuclear reaction) on remote targets in the atmosphere has to be forgotten in the current context. On the other hand, if the beam profile is so smooth that only a single filament will persist while the peak power is increased significantly to many times the critical power for self-focusing, the diameter and, hence, the volume of the filament will increase while the intensity inside this larger volume will still be clamped. Also, multiple re-focusing will take place. In practice, this is a tough condition to fulfill because any little fluctuation in intensity on the beam profile will lead to local self-focusing so long as the local power is higher than the critical power for self-focusing. This again results in multiple filaments. Furthermore, because the intensity is almost constant, any interaction making use of or sampling the filament core will result in a very stable outcome. One example is third harmonic generation [24].

The clamped intensity in air (or gases) is independent of pressure. Thus, when filamentation occurs at a high altitude in the atmosphere, the clamped intensity is always the same as that at sea level because when intensity clamping occurs, the nonlinear Kerr index change and the index due to plasma generation are equal: $n_2 I = \frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$. Both n_2 and $N_e(t)$ are linearly proportional to the gas density since $N_e(t)$ comes from tunnel ionization of the individual molecules. Hence, the gas density cancels out on the two sides of the equation, leaving behind an equation for the solution of the same clamped intensity I at any pressure.

IV. SUMMARY

The physical concepts underlying femtosecond laser filamentation are discussed. These include slice-by-slice self-focusing, intensity clamping, a background reservoir, pulse self-compression, and some consequences. Some possible misconceptions, such as the possibility of increasing the intensity indefinitely inside the self-focal volume and the self-transformation of the pulse into a line of light (filament), are dispelled.

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