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Phase aspect in photon emission and absorption

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In the literature one finds several conflicting accounts of the phase difference of stimulated and spontaneous emission, as well as absorption, with respect to an existing (triggering) electromagnetic field. One of these approaches proposes that stimulated emission and absorption occur in phase and out of phase with their driving field, respectively, whereas spontaneous emission occurs under an arbitrary phase difference with respect to an existing field. It has served as a basis for explaining quantum-mechanically the laser linewidth, its narrowing by a factor of 2 around the laser threshold, as well as its broadening due to amplitude–phase coupling, resulting in Henry's α-factor. Assuming the validity of Maxwell's equations, all three processes would, thus, violate the law of energy conservation. In semi-classical approaches, we investigate stimulated emission in a Fabry–Perot resonator, analyze the Lorentz oscillator model, apply the Kramers–Kronig relations to the complex susceptibility, understand the summation of quantized electric fields, and quantitatively interpret emission and absorption in the amplitude–phase diagram. In all cases, we derive that the phase of stimulated emission is 90° in lead of the driving field, and the phase of absorption lags 90° behind the transmitted field. Also spontaneous emission must obey energy conservation, hence it occurs with 90° phase in lead of an existing field. These semi-classical findings agree with recent experimental investigations regarding the interaction of attosecond pulses with an atom, thereby questioning the physical explanation of the laser linewidth and its narrowing or broadening.

OCIS codes: (300.2140) Emission; (260.3160) Interference; (260.5740) Resonance; (140.3460) Lasers; (140.3410) Laser resonators; (140.3430) Laser theory.

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1. INTRODUCTION

In his centennial paper [1], Einstein exploited a semi-classical rate-equation approach including the rates of stimulated and spontaneous emission and absorption to provide the physical foundation of Planck's law of blackbody radiation [2], thereby predicting the existence of stimulated emission, which was confirmed experimentally in 1928 [3]. By assuming conservation of energy and momentum, Einstein showed that an incident electromagnetic field at frequency ν triggers a two-level atom in its excited state to emit an additional electromagnetic field with an energy $h\nu$ that equals the energy gap between the two levels, such that the energy of the incident field increases by this energy during the interaction. Only a very small energy mismatch occurs due to recoil of the atom. The emitted field has the same frequency, the same direction, and the same polarization as the incident field, hence it is emitted into the same optical mode. In addition, spontaneous emission into the same optical mode occurs. In his original work, Einstein did not specify the phase difference between the incident and emitted electromagnetic field.

In the literature, three different accounts of this phase difference can be found. (i) The semi-classical Lorentz oscillator model [4] predicts that stimulated emission is in quadrature, i.e., 90° out of phase with the incident field [5–8]. (ii) The amplitude–phase diagram of Fig. 1(a) proposes that stimulated emission is in phase with the incident field (red solid arrows), whereas according to Lax [9], Haken [10], Henry [11], and others spontaneous emission occurs at an arbitrary phase angle θ with the incident field (red dashed-dotted arrow). (iii) Quantum-optically stimulated and spontaneous emission are both described by the same creation operator [16], hence either both processes must occur with the same phase difference, or the phase difference is not explicitly considered when applying the creation operator.

Despite their obvious incompatibility concerning the phase aspect, all three models have been applied to understand important optical phenomena. Model (i) has been exploited to derive the Kramers–Kronig relations [17,18] between susceptibility and absorption. In a simple rate-equation approach equivalent to Einstein's [1] without phase considerations, i.e., in line with model (iii), the power behavior of semiconductor lasers around the laser threshold has been described [19–22]. A combination of models (iii) and (ii) has been applied to calculate quantum–mechanically the fundamental laser linewidth and its reduction compared to the Schawlow–Townes linewidth [12] by an additional factor of 2 around laser threshold [9,10,13–15]. Model (ii) has served to justify this reduction of laser linewidth around the laser threshold [9–11]. According to Fig. 1(a), spontaneous

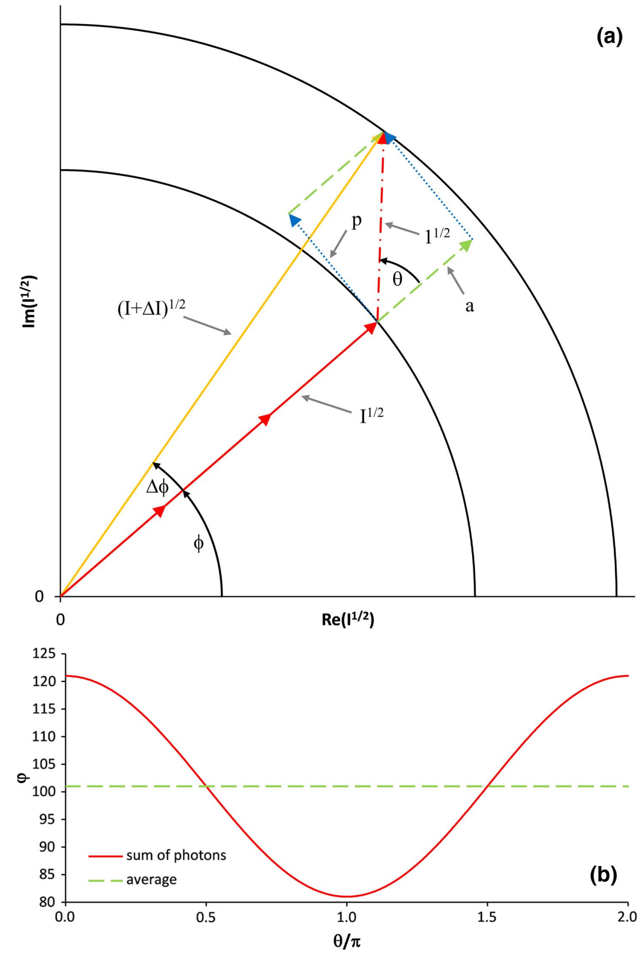

Fig. 1. (a) Amplitude–phase diagram visualizing the interpretation of quantum noise and laser linewidth by Lax [9], Haken [10], and Henry [11]. Quantum noise in a laser is said to be induced by adding with an arbitrary phase difference θ a spontaneously emitted photon (red dashed–dotted arrow) of intensity 1 to the intra-cavity laser field (red solid arrows) of intensity I and phase ϕ, resulting in an intra-cavity laser field of intensity $I + \Delta I$ (orange solid arrow) and inducing a phase shift $\Delta\phi$. Below laser threshold, all phase differences θ are proposed to generate noise, whereas, above laser threshold, amplitude fluctuations a ($\theta = 0$ or π, i.e., the projection of noise onto the direction of the green dashed arrow) are rapidly damped out by relaxation oscillations, and only phase fluctuations p ($\theta = \pm\pi/2$, i.e., the projection of noise onto the direction of the blue dotted arrow) contribute to noise, thereby reducing the laser linewidth compared to the Schawlow–Townes linewidth [12] by a factor of 2 [9–11,13–15]. (b) Number φ of photons resulting from the interference according to Eq. (1) between one photon and 100 photons (red solid curve) versus phase difference θ and medium of 101 photons averaged over all θ (green dashed line).

emission induces amplitude and phase fluctuations (projection onto the axes with $\theta_a = 0$–$180°$ and $\theta_p = \pm 90°$, respectively), of which the latter constitute the quantum noise that determines the fundamental laser linewidth, whereas the former are damped out in a laser, thereby reducing the laser linewidth by a factor of 2. It has also served to derive Henry's α-factor, which quantifies broadening of the laser linewidth due to amplitude–phase coupling via the refractive index [11].

Since these three incompatible versions seem to describe various optical phenomena, does then the phase aspect matter at all? The arguments presented in this paper suggest that the phase is of fundamental importance, and, consequently, there must not occur an ambiguity concerning the phase difference. By exploiting Maxwell's equations and the law of energy conservation, investigating stimulated emission in a Fabry–Perot resonator, analyzing the Lorentz oscillator model, applying the Kramers–Kronig relations to the complex susceptibility, understanding the summation of quantized electric fields, and quantitatively interpreting emission and absorption in the amplitude–phase diagram, we derive a consistent semi-classical picture of the phase aspect in stimulated and spontaneous emission, as well as absorption. How far a quantum-optical treatment can confirm or conflicts with this picture is not a subject of this paper, but is currently under investigation.

2. SEMI-CLASSICAL VERSUS QUANTUM-OPTICAL DESCRIPTION

Emission and absorption of a photon are quantized processes. When judging the present work from a quantum-optical point of view, one should keep in mind the following points. (i) Only on a sufficiently short time scale can a process violate the law of energy conservation according to the uncertainty principle [23,24]. At longer time scales, the law of energy conservation applies in quantum mechanics. (ii) The classical Maxwell equations [25] maintain their full validity in quantum optics, but with the additional requirement of a quantization of optical energy, as was demonstrated by Dirac [26]. (iii) The amplitude–phase diagram is not a sloppy way of sketching some processes, but can—and should—be understood as a quantitative vectorial description. (iv) Einstein's semi-classical rate-equation approach [1] not only confirmed Planck's law [2], but also delivered the Einstein A and B coefficients of spontaneous and stimulated emission, which were found entirely consistent with a full quantum-mechanical treatment; see, e.g., Schiff [27]. (v) The Kramers–Kronig relations [17,18] are bidirectional mathematical relations between the real and the imaginary part of any complex function that is analytic in the upper half of the complex plane. The analyticity condition is a consequence of causality in physical systems. Consequently, the Kramers–Kronig relations apply to the complex susceptibility, semi-classically as well as quantum-optically. (vi) The Lorentz oscillator model [4] is an approximation to the quantum theory that is equivalent to the standard perturbation-theoretical approach to absorption and stimulated emission; see, e.g., Schiff [28], in which a weak oscillator strength is assumed, such that the transition rate is essentially constant and the atom is still in its initial state (upper or lower state of the transition) after some time interval that is long compared to the oscillation period (Fermi's golden rule [29], originally formulated by Dirac [26]). (vii) Despite its obvious crudeness, the Lorentz oscillator model has enormous prediction power in optics, as was pointed out by Feynman *et al.* [30] and Weisskopf [31]. (viii) Based upon the Lorentz oscillator model, stimulated emission and absorption may be understood in the semi-classical approach of treating the atom, which in the simplest case is considered a two-level system, quantum-mechanically and the electromagnetic field, which is in near resonance with the transition between the two atomic levels, classically, as was emphasized by Milonni *et al.* [6,7]. Therefore, it is not *a priori* clear that semi-classical models insufficiently describe quantized emission and absorption processes.

3. MAXWELL'S EQUATIONS, INTERFERENCE, AND CONSERVATION OF ENERGY

In the following, by use of the term "photon" we will solely refer to the fundamental energy unit $h\nu$ that corresponds to the quantized energy of one photon, but we will not associate any quantum-statistical properties with it. The parameter φ represents the number of these energy units called photons that is present in a classical electromagnetic field (or the expectation value of a quantum-optical coherent state, if you will). In principle, φ may assume any non-integer value. However, it is probably easy to agree upon the fact that the conservation of energy requires that an emission or absorption process by a two-level atom changes the value of φ by ± 1 energy unit $h\nu$.

Based upon Maxwell's equations [25], superposition of two co-propagating electromagnetic waves at the same frequency ν, with electric-field amplitudes E_1 and E_2 and a phase difference θ, yields the intensity:

$$\varphi \propto I_{1+2} = \frac{c\varepsilon_0}{2}|\vec{E}_1 + \vec{E}_2|^2 = \frac{c\varepsilon_0}{2}[|E_1|^2 + 2|E_1 E_2|\cos(\theta) + |E_2|^2]. \tag{1}$$

ε_0 is the vacuum permittivity and c is the speed of light in the medium of refractive index n. For $\cos(\theta) \neq 0$, the interference term does not vanish and the law of energy conservation is obviously violated. Two fields with $E_1 = E_2$ constructively (destructively) interfere to produce four (zero) times the intensity of each field alone. When adding a field representing one photon ($E_2 \propto 1^{1/2} = 1$) to a field representing 100 photons ($E_1 \propto 100^{1/2} = 10$), then averaged over all phase angles, $\overline{\cos}(\theta) = 0$, the expected energy of 101 photons emerges, whereas constructive (destructive) interference yields the energy of 121 (81) photons; see Fig. 1(b). Nevertheless, energy can be conserved, if there is a concrete source or drain of energy in the system, for example, enhanced scattering of a perpendicular external light source into a resonant cavity [32]. In the absence of such a source or drain, energy can be conserved only if the opposite interference occurs in another location of the optical system, i.e., constructive or destructive interference never occurs alone!

Let us analyze the processes in Fig. 1(a) with respect to energy conservation. If stimulated emission occurred in phase with the incident field, $\cos(\theta) = 1$, then according to Eq. (1) each stimulated-emission event would generate an excess of photons, thereby violating the law of energy conservation. If spontaneous emission occurred at an arbitrary phase angle, then, according to Eq. (1), each spontaneous-emission event would either generate or annihilate extra photons [Fig. 1(b)], and in the amplitude–phase diagram [Fig. 1(a)] the added intensity ΔI would not correspond to the intensity generated by one photon. As pointed out by Henry [11], only when averaging over many spontaneous-emission events, $\overline{\cos}(\theta) = 0$, would energy be conserved [Fig. 1(b)]. In a lasing resonator, such a violation of energy conservation by stimulated and spontaneous emission could be experimentally manifested by cavity dumping of the stored optical energy within a single resonator round trip. Since energy must be conserved, this interpretation of stimulated and spontaneous emission is obviously questionable. The assumptions by quite many scientists that the electric field magically adjusts its magnitude such that the energy is conserved or that the energy difference magically comes out of or dissipates into the universe can safely be discarded.

4. STIMULATED EMISSION IN A FABRY–PEROT RESONATOR

The problem of energy conservation manifests itself in a fundamentally important optical system, the Fabry–Perot resonator. In Fig. 2(a), a monochromatic external light source continuously launches light into a resonator whose only losses are the outcoupling losses through its two mirrors. A steady state of light launched into, circulating inside, and emitted from the resonator is established, described by the Airy distributions [33]

$$\begin{aligned} A_{\text{circ}} &= I_{\text{circ}}/I_{\text{laun}} = \frac{1}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2(\Delta\phi_{\text{RT}}/2)}, \\ A'_{\text{trans}} &= I_{\text{trans}}/I_{\text{inc}} = (1-R_1)(1-R_2)A_{\text{circ}}, \\ A'_{\text{back}} &= I_{\text{back}}/I_{\text{inc}} = (1-R_1)^2 R_2 A_{\text{circ}}, \\ A'_{\text{refl}} &= I_{\text{refl}}/I_{\text{inc}} \\ &= \left[\left(\sqrt{R_1} - \sqrt{R_2}\right)^2 + 4\sqrt{R_1R_2}\sin^2(\Delta\phi_{\text{RT}}/2)\right]A_{\text{circ}}, \\ A'_{\text{trans}} + A'_{\text{refl}} &= (I_{\text{trans}} + I_{\text{refl}})/I_{\text{inc}} = 1, \end{aligned} \tag{2}$$

where r_i and $R_i = |r_i|^2$ are the amplitude and intensity reflectivity of mirror i, respectively. $\Delta\phi_{\text{RT}}$ is the phase shift accumulated over one round trip. The different electric fields are displayed in Fig. 2(a); their intensities are $I \propto |E|^2$, and their spectral dependencies are given by the Airy distributions A with respect to the launched intensity or A' with respect to the incident intensity [33]. For arbitrary $\Delta\phi_{\text{RT}}$, the energy is conserved, $A'_{\text{trans}} + A'_{\text{refl}} = 1$, because the interference between E_{RT} and E_{laun} is compensated by the opposite interference between $E_{\text{refl},1}$ and E_{back}.

Now we move the light source into the resonator [Fig. 2(b)]. A light source inside the mode must be transparent in order not to block the propagating light; consequently, the propagating light will interact with the light source. Let us assume that the light source is a pumped inverted medium (an atomically thin gain sheet placed close to mirror 1, oriented perpendicular to the resonator axis) that continuously generates E_{gen} via stimulated emission triggered by E_{RT}, resulting in the combined field E_{circ}. For simplicity, we neglect spontaneous emission and assume that several stimulated-emission processes can occur

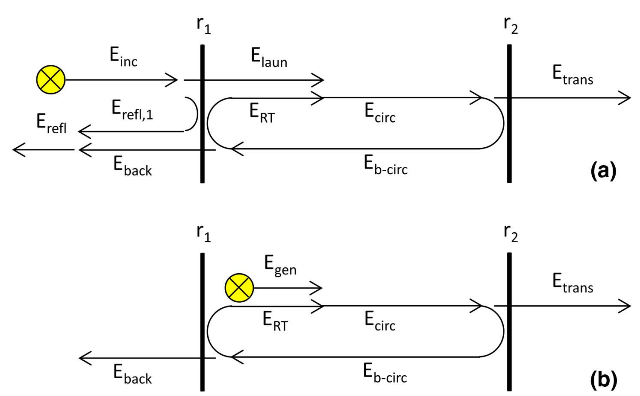

Fig. 2. Schematic of a Fabry–Perot resonator and the relevant electric fields E for (a) light launched from outside [33] and (b) light generated inside the resonator.

simultaneously in different lateral regions of the gain sheet without influencing each other. Furthermore, for comparison with Fig. 2(a), we assume a race-track resonator with unidirectional light propagation, such that the backward-circulating field $E_{b\text{-circ}}$ does not penetrate the active medium and the stimulated-emission process is uni-directional. If stimulated emission occurred "in phase" and we pumped the medium to a desired inversion, such that in Figs. 2(a) and 2(b) the same field $E_{\rm gen} = E_{\rm laun}$ interfered constructively with $E_{\rm RT}$ at the resonance frequency ν_q, then the same steady state of light generated, circulating inside, and emitted from the resonator would be established and the same fields $E_{\rm trans}$ and $E_{\rm back}$ would be emitted in both situations. However, since in Fig. 2(b) $E_{\rm back}$ cannot destructively interfere, because there is no $E_{\rm refl,1}$, the constructive interference is not compensated for. Therefore, if the phase shift $\Delta\phi_{\rm em}$ potentially induced by stimulated emission between $E_{\rm RT}$ and $E_{\rm circ}$ equals zero, energy is not conserved:

$$\begin{aligned} A_{\rm circ} &= I_{\rm circ}/I_{\rm gen} \\ &= \frac{1}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2[(\Delta\phi_{\rm RT}+\Delta\phi_{\rm em})/2]}, \\ A_{\rm trans} &= I_{\rm trans}/I_{\rm gen} = (1-R_2)A_{\rm circ}, \\ A_{\rm back} &= I_{\rm back}/I_{\rm gen} = (1-R_1)R_2A_{\rm circ}, \\ A_{\rm emit} &= A_{\rm trans} + A_{\rm back} = (I_{\rm trans}+I_{\rm back})/I_{\rm gen} = (1-R_1R_2)A_{\rm circ}, \\ A_{\rm emit} &> 1 \quad \text{for } \Delta\phi_{\rm RT} = \Delta\phi_{\rm em} = 0. \end{aligned} \tag{3}$$

$A_{\rm trans}$ and $A_{\rm back}$ are the Airy distributions of $I_{\rm trans}$ and $I_{\rm back}$, respectively, with respect to $I_{\rm gen}$. Their sum $A_{\rm emit}$ is displayed in Fig. 3(a) as a function of $(R_1R_2)^{1/2}$. Since light builds up inside the resonator and stimulates emission around the resonance frequency ν_q (either in a broadband gain medium or by tuning the resonator length, such that the resonance frequency ν_q coincides with the emission frequency), $\Delta\phi_{\rm RT}$ becomes a multiple of 2π. Consequently, the phase shift indicated in the legend of Fig. 3(a) is solely due to $\Delta\phi_{\rm em}$. Energy conservation requires $A_{\rm emit} = 1$ (black line).

If stimulated emission occurred in phase, $\Delta\phi_{\rm em} = 0$ [solid gray curve in Fig. 3(a)], the law of energy conservation would be violated; e.g., for $R_1 = R_2 = 0.7$, $A_{\rm emit} = 5.7$ times the light generated by stimulated emission would be emitted through both mirrors. Also, for all phase shifts $\Delta\phi_{\rm em} \neq 0$ energy is not conserved, except for one specific value of $\Delta\phi_{\rm em}$ for each value of $(R_1R_2)^{1/2}$; see Fig. 3(a).

Assuming resonance, i.e., $\Delta\phi_{\rm RT}$ is a multiple of 2π, to ensure energy conservation, i.e., $A_{\rm emit} = 1$, the condition

$$\begin{aligned} &(1-R_1R_2)A_{\rm circ} = 1 \\ &\Rightarrow 2\sin^2(\Delta\phi_{\rm em}/2) = 1-\sqrt{R_1R_2} \Rightarrow \cos(\Delta\phi_{\rm em}) = \sqrt{R_1R_2} \\ &\Rightarrow \tan(\Delta\phi_{\rm em}) = \sqrt{\frac{1-\cos^2(\Delta\phi_{\rm em})}{\cos^2(\Delta\phi_{\rm em})}} = \sqrt{\frac{1-R_1R_2}{R_1R_2}} \end{aligned} \tag{4}$$

must be fulfilled; see Fig. 3(b). The phase shift $\Delta\phi_{\rm em}$ induced by stimulated emission differs from zero and depends on R_1 and R_2. We convert this reflectivity dependence to a photon dependence. Since each intensity is proportional to the corresponding photon number, the ratio between the numbers $\varphi_{\rm RT}$ of photons triggering stimulated emission and $\varphi_{\rm gen}$ of photons generated by stimulated emission equals the Airy distribution $A_{\rm RT}$, from which we derive the phase shift:

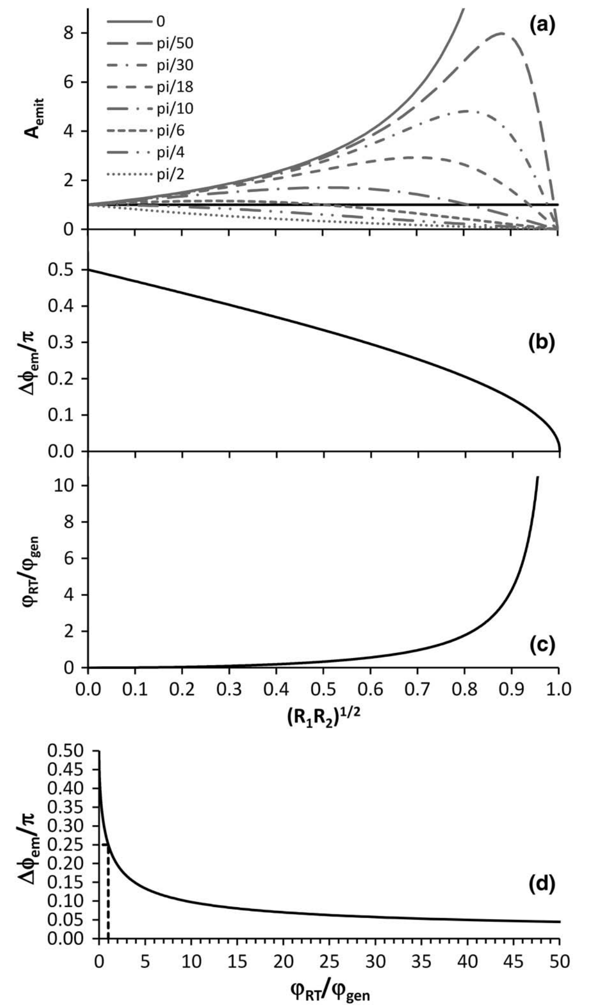

Fig. 3. (a) Requirement of energy conservation, $A_{\rm emit} = 1$ (black line), and violation of energy conservation for different potential phase shifts $\Delta\phi_{\rm em} = 0, \pi/50, \pi/30, \pi/18, \pi/10, \pi/6, \pi/4$, and $\pi/2$ (see legend) from the field $E_{\rm RT}$ to the field $E_{\rm circ}$ induced by interference of $E_{\rm RT}$ with the field $E_{\rm gen}$ generated by stimulated emission, (b) the phase shift $\Delta\phi_{\rm em}$ that is required to obtain energy conservation, and (c) the ratio $\varphi_{\rm RT}/\varphi_{\rm gen}$ of triggering photon number $\varphi_{\rm RT}$ over generated photon number $\varphi_{\rm gen}$ as a function of $(R_1R_2)^{1/2}$. (d) Phase shift $\Delta\phi_{\rm em}$ induced by stimulated emission as a function of the ratio $\varphi_{\rm RT}/\varphi_{\rm gen}$. For $\varphi_{\rm RT}/\varphi_{\rm gen} = 1$ one obtains $\Delta\phi_{\rm em} = \pi/4$ (dashed lines).

$$\begin{aligned} \varphi_{\rm RT}/\varphi_{\rm gen} &= I_{\rm RT}/I_{\rm gen} = A_{\rm RT} = R_1R_2A_{\rm circ} \\ &= \frac{R_1R_2}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2[(\Delta\phi_{\rm RT}+\Delta\phi_{\rm em})/2]} \\ &= \frac{R_1R_2}{1-R_1R_2} \\ &\Rightarrow \tan(\Delta\phi_{\rm em}) = \sqrt{\varphi_{\rm gen}/\varphi_{\rm RT}}, \end{aligned} \tag{5}$$

where we used $\Delta\phi_{\rm RT} = 0$ and Eq. (4). The ratio $\varphi_{\rm RT}/\varphi_{\rm gen}$ is displayed in Fig. 3(c) and the final result of Eq. (5) is shown in Fig. 3(d). Only for an infinite ratio $\varphi_{\rm RT}/\varphi_{\rm gen}$ is the induced phase shift $\Delta\phi_{\rm em}$ zero. The smaller the number of photons triggering stimulated emission and the larger the number of photons generated, the further the induced phase shift $\Delta\phi_{\rm em}$ increases to $\pi/2$. If one photon triggers stimulated emission of one photon, then $\Delta\phi_{\rm em} = \pi/4$. Since the second angle in the vector diagram comprising the fields $E_{\rm RT}$, $E_{\rm gen}$, and $E_{\rm circ}$ is also known, the phase difference between triggering and stimulated field equals

$$\theta = \pi - \arctan\sqrt{\varphi_{\rm gen}/\varphi_{\rm RT}} - \arctan\sqrt{\varphi_{\rm RT}/\varphi_{\rm gen}} = \pi/2. \quad (6)$$

In resonance, stimulated emission occurs with a 90° phase difference between driving and generated field, its direct consequence being the phase shift $\Delta\phi_{\rm em}$ of Eq. (5) between driving and transmitted field.

For this derivation we have only assumed the validity of Maxwell's equations and the law of energy conservation. Although the curve in Fig. 3(d) is continuous, one can easily impose a quantization of energy by allowing only integer values of $\varphi_{\rm RT}$ and $\varphi_{\rm gen}$ in the ratio $\varphi_{\rm RT}/\varphi_{\rm gen}$.

5. LORENTZ OSCILLATOR MODEL AND KRAMERS–KRONIG RELATIONS

The Lorentz oscillator model describes the motion of electrons with electric charge e and mass m_e, bound as a cloud with electron density N_e within an atom, as a damped harmonic oscillation with an angular resonance frequency ω_0 and gain/damping rate constant γ_e (positive for stimulated emission, negative for absorption) displaced by a distance x from its rest position by an external driving electric field $E_{\rm ext}$ oscillating with angular frequency $\omega_{\rm ext}$. The magnetic force is neglected. Its mathematical treatment is equivalent to that of a mechanical spring oscillator. The linear second-order ordinary differential equation of motion is solved, yielding the atomic polarization P_e and phase difference θ between the driving electric field and the polarization:

$$m_e\ddot{x}(t) + 2\gamma_e m_e\dot{x}(t) + \omega_0^2 m_e x(t) = -eE_{\rm ext}\exp(-i\omega_{\rm ext}t),$$

$$P_e = -N_e e x(t) = \frac{N_e e^2/m_e}{\omega_0^2 - \omega_{\rm ext}^2 - i2\gamma_e\omega_{\rm ext}} E_{\rm ext}\exp(-i\omega_{\rm ext}t + \theta),$$

$$\tan(\theta) = \frac{2\gamma_e\omega_{\rm ext}}{\omega_0^2 - \omega_{\rm ext}^2}. \quad (7)$$

The same phase difference θ as from the Lorentz oscillator model in Eq. (7) obtains from the Kramers–Kronig relations [17,18] between the real part (susceptibility) χ_e' and the imaginary part (gain or absorption) χ_e'' of the complex susceptibility χ_e [Figs. 4(a)–4(d)]:

$$P_e = \varepsilon_0\chi_e E_{\rm ext}\exp(-i\omega_{\rm ext}t + \theta),$$

$$\chi_e = \chi_0\frac{\omega_0^2}{\omega_0^2 - \omega_{\rm ext}^2 - i2\gamma_e\omega_{\rm ext}},$$

$$\chi_0 = \frac{N_e e^2}{\varepsilon_0 m_e\omega_0^2},$$

$$\chi_e' = \chi_0\frac{-(\omega_0^2 - \omega_{\rm ext}^2)\omega_0^2}{(\omega_0^2 - \omega_{\rm ext}^2)^2 + (2\gamma_e\omega_{\rm ext})^2},$$

$$\chi_e'' = \chi_0\frac{2\gamma_e\omega_{\rm ext}\omega_0^2}{(\omega_0^2 - \omega_{\rm ext}^2)^2 + (2\gamma_e\omega_{\rm ext})^2},$$

$$\tan(\theta) = \frac{\chi_e''}{\chi_e'} = \frac{2\gamma_e\omega_{\rm ext}}{\omega_0^2 - \omega_{\rm ext}^2}. \quad (8)$$

As is well known from mechanical oscillators, when the driving frequency $\omega_{\rm ext}$ is significantly lower (higher) than the resonance frequency ω_0, the oscillation is in (out of) phase with the driving

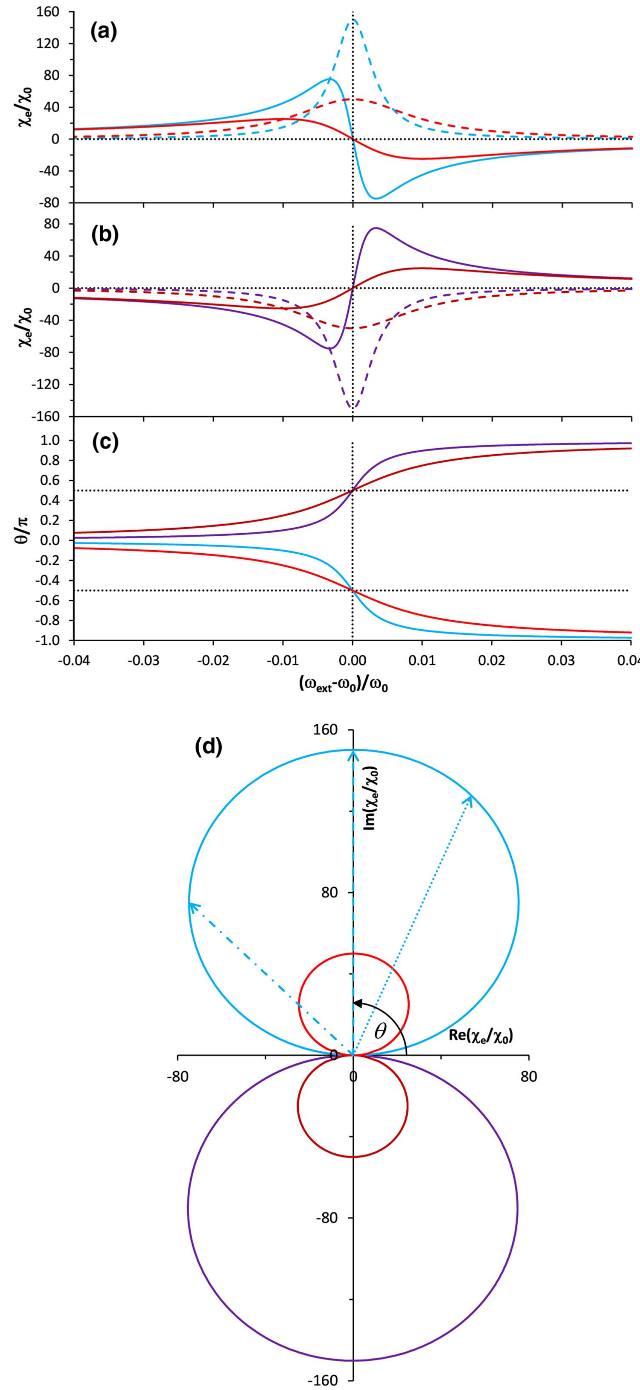

Fig. 4. Real part χ_e' (solid lines) and imaginary part χ_e'' (dashed lines) of the susceptibility, calibrated to χ_0, for $\gamma_e = \pm 0.00333\omega_0$ (blue curves) and $\gamma_e = \pm 0.01\omega_0$ (red curves) in (a) stimulated emission and (b) absorption. (c) Phase difference θ between the amplitudes of a driven atomic oscillator and its driving electric field as a function of driving frequency. (d) Complex susceptibility χ_e, calibrated to χ_0, as a function of $\omega_{\rm ext}$, for the four examples displayed in (a) and (b). For the examples of $\gamma_e = \pm 0.00333\omega_0$, the arrows indicate the situations of $(\omega_{\rm ext} - \omega_0)/\omega_0 = -1.38 \times 10^{-3}$, 0 (resonance), and 3.34×10^{-3}, resulting in $\theta = 3/8\pi$ (dotted arrow), $\pi/2$ (resonance, dashed arrow), and $3/4\pi$ (dashed–dotted arrow), respectively. The phase difference in resonance of $\theta = \pi/2$ is indicated by the black curved arrow, which points in the direction of increasing $\omega_{\rm ext}$.

field. In resonance, the phase difference θ crosses the value of $\pi/2$; see Fig. 4(c). Since the electric field scales with the distance between the two charges of the oscillating dipole, it is generated in phase with the dipole oscillation, and stimulated emission is 90° in lead of the triggering field. Likewise, in resonant absorption the phase difference between the atomic oscillation and the transmitted field is $\theta = -\pi/2$, i.e., the atomic oscillation lags 90° behind. This simple derivation confirms that in stimulated emission the emissive part of the generated dipole field is in quadrature with the driving electromagnetic field [5–8].

Consequently, in a resonant stimulated-emission process, the interference term in Eq. (1) vanishes, the two individual intensities add up, and the energy is conserved. For exactly this reason, (i) Einstein was allowed to neglect interference in his semi-classical rate-equation derivation [1] of Planck's law [2] and (ii) laser performance can be described—and important laser parameters, such as threshold and slope efficiency, can be obtained—with a distributed-intensity-gain coefficient g by a differential rate equation that calculates the photon number φ but neglects interference:

$$\frac{d}{dt}\varphi = R_{st} - R_{\text{decay}} = cg\varphi - \frac{1}{\tau_c}\varphi. \tag{9}$$

R_{st} and R_{decay} are the stimulated-emission and photon-decay rates, respectively, and τ_c is the photon-decay time. Also, the quantum-optical laser master equation neglects interference [34].

6. AMPLITUDE–PHASE DIAGRAM AND QUANTIZED ELECTRIC FIELDS

For φ_{ext} photons triggering the emission of $\varphi_{\text{gen}} = 1$ photon, the situation of $\theta = \pi/2$ is illustrated in the amplitude–phase diagram of Fig. 5(a). The resulting phase shift $\Delta\phi_{\text{em}}$ between the incident and transmitted electric field is

$$\tan(\Delta\phi_{\text{em}}) = \sqrt{1/\varphi_{\text{ext}}}. \tag{10}$$

Assuming an incident electromagnetic field containing the energy of $\varphi_{\text{ext}} = 1$ photon, the build-up of a larger electromagnetic field by consecutive stimulated emission of electromagnetic fields, each containing the energy of $\varphi_{\text{gen}} = 1$ photon, is displayed in Fig. 5(b). The total phase shift accumulated by the consecutive stimulated emission of $n - 1$ photons by one initial photon with an arbitrary phase, resulting in an electromagnetic wave containing n photons, amounts to

$$\Delta\phi_n = \sum_{i=1}^{n-1} \Delta\phi_{\text{em},i} = \sum_{i=1}^{n-1} \arctan\left(1/\sqrt{i}\right) \quad \text{for } n \geq 1. \tag{11}$$

The total phase shift of Eq. (11) establishes a relation among all these states.

Simultaneous independent stimulated emission of several photons, $\varphi_{\text{gen}} > 1$, induces a phase shift $\Delta\phi_{\text{em}}$ in Eq. (5) that is smaller than the phase shift $\Delta\phi_n$ of Eq. (11) accumulated by consecutive stimulated emission of single photons (Fig. 6):

$$\Delta\phi_{\text{em}}(\varphi_{\text{em}} > 1) < \Delta\phi_n(n = \varphi_{\text{em}} > 1). \tag{12}$$

For investigating the Fabry–Perot resonator above, we chose an atomically thin gain sheet and assumed a simultaneous independent stimulated emission of several photons into the same mode. It is an interesting question whether this assumption is physically justified. If true, the total phase shift induced when building up a light beam depends on the way the photons are generated, consecutively or simultaneously. If not true, i.e., the simultaneous emission of several photons within the same mode is correlated, such that these photons must obey the law of energy conservation also with respect to each other, then the total phase shift of Eq. (11) establishes a unique relation among all photon numbers φ. Such a correlation could lead to the phenomenon of superradiance [35–37].

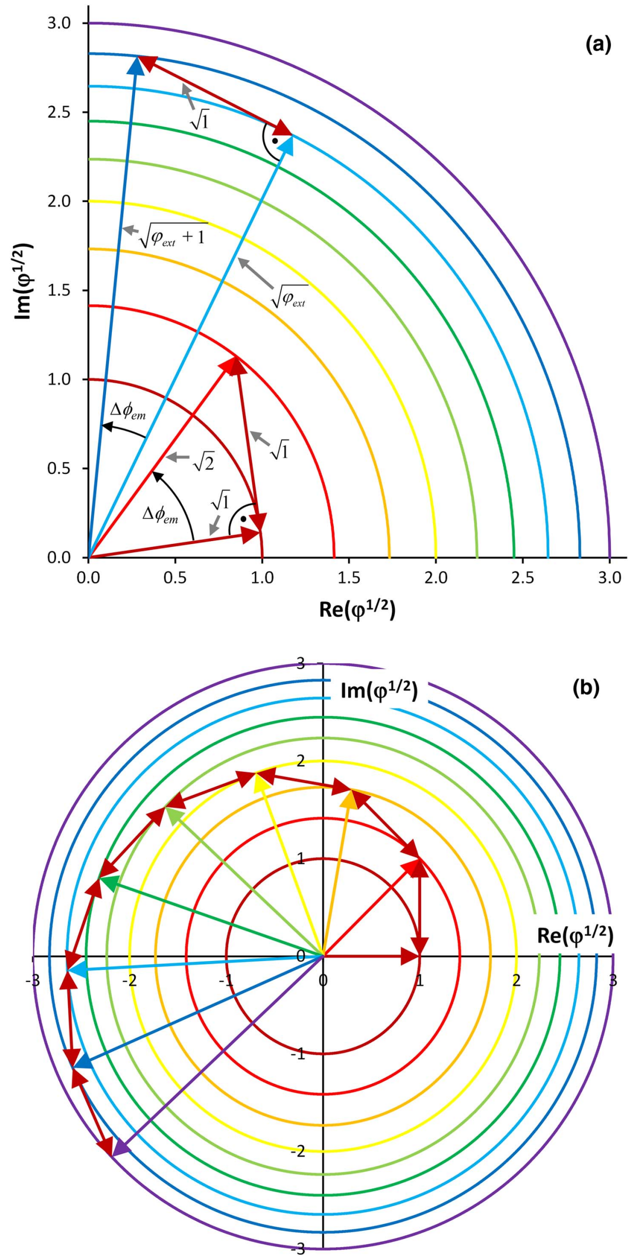

Fig. 5. (a) Quadrant of the amplitude–phase diagram illustrating the process of stimulated emission (with the dark-red arrows of the emitted field pointing toward the upper left): a field of φ_{ext} photons triggers an atom in its excited state to emit $\varphi_{\text{em}} = 1$ photon, in the two situations of (i) $\varphi_{\text{ext}} = 1$ and (ii) $\varphi_{\text{ext}} = 7$. In both situations, the indicated right angle is $90° = 180° - \theta$, hence $\theta = 90°$. The color code denotes the amplitude in units of $\varphi^{1/2}$, from $\varphi = 1$ photon (dark red) to $\varphi = 9$ photons (violet). The same diagram holds true for absorption (with the dark-red arrows of the absorbed field pointing toward the lower right). (b) Build-up of a light beam by the consecutive addition of single photons in the amplitude–phase diagram.

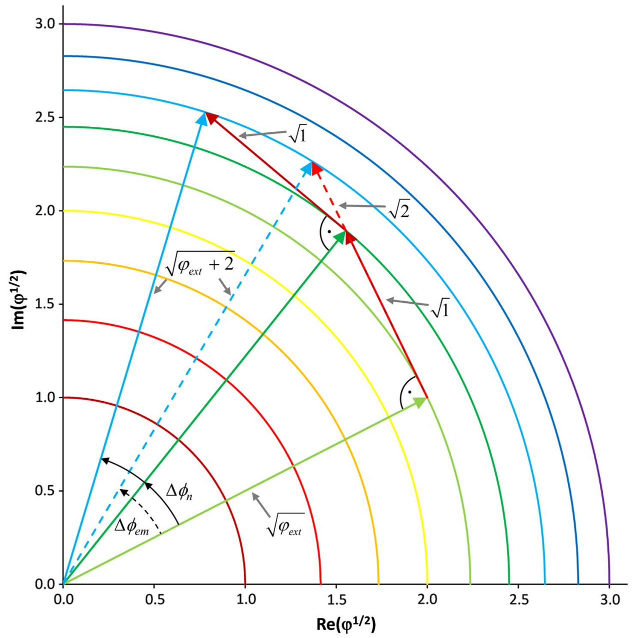

Fig. 6. Quadrant of the amplitude–phase diagram comparing the simultaneous, independent addition of several photons (dashed arrows; here, a field of $\varphi_{\mathrm{ext}} = 5$ photons triggers atoms in their excited state to simultaneously emit $\varphi_{\mathrm{em}} = 2$ photons), resulting in a phase shift $\Delta\phi_{\mathrm{em}}$, with the consecutive addition of two single photons (solid arrows), resulting in an accumulated phase shift $\Delta\phi_n > \Delta\phi_{\mathrm{em}}$.

One should not fall into the trap of believing that this relative phase shift might be arbitrary. Of course, there is an arbitrary starting phase. By rotating the coordinate system by an appropriate fixed amount, the starting phase is set to zero in Fig. 5(b), whereas, in the examples of Fig. 5(a), it is assumed different from zero. In addition, the coordinate system can be rotated time-dependently. In fact, in Fig. 5 the real and imaginary axes are rotated by the oscillatory term $e^{i\omega t}$, so that the displayed arrows that rotate with this angular speed stand still in the graph. Beyond that, no further freedom exists in this semi-classical treatment.

When an electromagnetic field that was generated by individual atomic emission processes triggers stimulated emission of another electromagnetic field by an atom, it requires only two ingredients for a correct quantization, namely, (i) each individual amplitude must be proportional to the square root $\sqrt{\varphi}$ of an integer photon number φ, and (ii) the phase difference must be $\theta = 90°$, the automatic consequence being the phase shift $\Delta\phi_{\mathrm{em}}$ of Eq. (5) between driving and transmitted field. An example of $\varphi_{\mathrm{ext}} = 16$ photons triggering the emission of $\varphi_{\mathrm{gen}} = 1$ photon is illustrated in Fig. 7(a), its result being quantitatively equivalent to the same process when displayed in the amplitude–phase diagram, as shown for other photon numbers in Fig. 5(a). The build-up of an electromagnetic field containing φ photons by consecutive stimulated emission of electromagnetic fields containing the energy of single photons is displayed in Fig. 7(b), its result being quantitatively equivalent to Fig. 5(b).

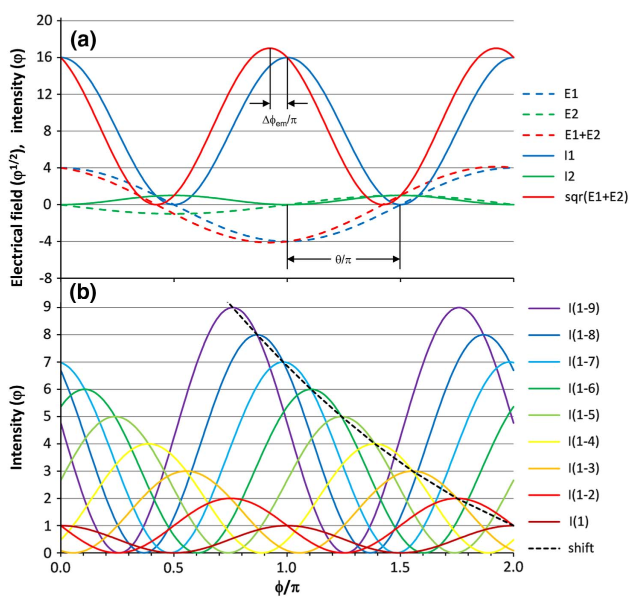

Fig. 7. (a) Example of the summation according to Eq. (1) of two electric fields of amplitudes equivalent to $\varphi^{1/2} = 4$ and 1, with a phase difference of $\theta = \pi/2$. The phase shift $\Delta\phi_{\mathrm{em}} = 0.1476\pi$ is equal to Eq. (10), and the energy is conserved, as $16 + 1 = 17$ photons emerge. (b) Consecutive addition of single photons to an existing electromagnetic field. Intensity of the light beam in units of φ. The black dashed line calculated from Eq. (11) indicates the phase shift $\Delta\phi_n = \Sigma(\Delta\phi_{\mathrm{em}})$ accumulated with increasing number φ of photons.

7. EXPERIMENTAL VERIFICATION

Very recently, the group of Ferenc Krausz found experimentally in the interaction of an attosecond pulse with an atom the following signatures [38]. When the relative phase of an atomic polarization is in lead of the electromagnetic field, energy is transferred from the atom to the field (stimulated emission), whereas when the phase of the atomic polarization lags behind the electromagnetic field, energy is transferred from the field to the atom (absorption). Considering that the emitted field occurs in phase with the atomic polarization, this experimental finding coincides with the semi-classical result that resonant stimulated emission is 90° in lead of the incident field, whereas absorption lags 90° behind the transmitted field. In a book review [39], Kastler also emphasized the necessity of a 90° phase difference in stimulated emission in order for the energy to be conserved and pointed toward an experimental verification of this phase difference by Meslin [40].

Very recent progress in another field dealing with resonant photonic systems, namely, plasmonic nanostructures, also clearly indicates that in resonance a 90° phase difference occurs, and that considering this phase shift is essential for the interpretation and understanding of experimental results [41]. Besides, a similar 90° phase difference is found in optical parametric amplification and oscillation [42,43], when energy is to be conserved. When the idler is generated by the signal (equivalent to stimulated emission of idler light or absorption of signal light), a 90° phase difference occurs between signal and idler. When the energy returns from the idler to the signal (equivalent to absorption of idler light or stimulated emission of signal light), the opposite 90° phase difference occurs, leaving the resulting idler and signal phases unchanged, nevertheless at a 90° phase difference with respect to each other. It is similar to the situation in Fig. 5(a), where the dark-red double arrows resembling stimulated emission and absorption occur at opposite 90° phase difference, leaving the phase of the electric field resulting from the two subsequent processes unchanged, nevertheless at a 90° phase difference with respect to the atomic polarization.

8. OPTICAL MODES

A mode is defined by its resonance frequency and spectral mode shape (which can be Lorentzian, but also highly distorted [33]), its transverse spatial mode shape (in an open two-mirror resonator these are the Hermite–Gaussian TEM_{xy} modes), and its polarization (in the simplest case, two linear polarizations). If all modes are orthogonal with each other in these three properties (which is not necessarily the case [44–47]), photons in the same mode share these three properties and trigger stimulated emission into this mode. In contrast, a mode is not distinguished from other modes by a unique phase, and light propagating in this mode can assume any phase value. Consequently, there is no need for an electromagnetic field triggered by stimulated emission to have the same phase as the triggering electric field. As we have reconfirmed above in multiple ways, semi-classically it must be in quadrature with the triggering field.

9. VACUUM FLUCTUATIONS AND SPONTANEOUS EMISSION

A vacuum fluctuation that occurs during a very short time scale can violate the law of energy conservation according to the uncertainty principle [23,24]. If vacuum fluctuations appear in an empty mode, they generate a time-averaged zero-point energy corresponding to half a photon. If an electromagnetic field exists in the mode, the vacuum field adds onto that field with an arbitrary phase difference θ.

Spontaneous emission is the consequence of vacuum fluctuations in the presence of optically active, excited species. In contrast to vacuum fluctuations, each individual spontaneous-emission event must increase the field amplitude and obey the law of energy conservation in the same manner as stimulated emission, because one atomic excitation is converted to an electromagnetic energy equivalent to one photon that survives at a macroscopic time scale. This statement agrees with the quantum-optical description of spontaneous emission, in which the creation operator $\hat{a}^\dagger$ increases the photon number by one. If in a spontaneous-emission event the photon is emitted into an unoccupied mode, it enters this mode under a phase that is determined solely by the triggering vacuum fluctuations. However, if the mode is occupied by an electromagnetic field, emission under an arbitrary phase difference θ with respect to the existing field would violate the law of energy conservation; see Fig. 1(b). Consequently, spontaneous emission must occur with the same phase difference of $\theta = \pi/2$ relative to the total field as stimulated emission.

Only under this condition was it possible for Einstein, by use of a semi-classical rate-equation approach [1] that neglects interference, to confirm Planck's law [2] and derive the Einstein A and B coefficients of spontaneous and stimulated emission. If spontaneous emission occurred with an arbitrary phase difference with respect to an existing electromagnetic field, his derivation would have had to take interference into account.

Here we present a physical picture of spontaneous and stimulated emission (Fig. 8) that is consistent with all aspects discussed above. Emission of a photon into a resonator mode takes longer than the resonator round-trip time [48]. At this time scale, the energy and phase fluctuations of the many extremely fast vacuum fluctuations average out to half a vacuum photon added to the existing field of φ_{ext} photons. This averaged total field of

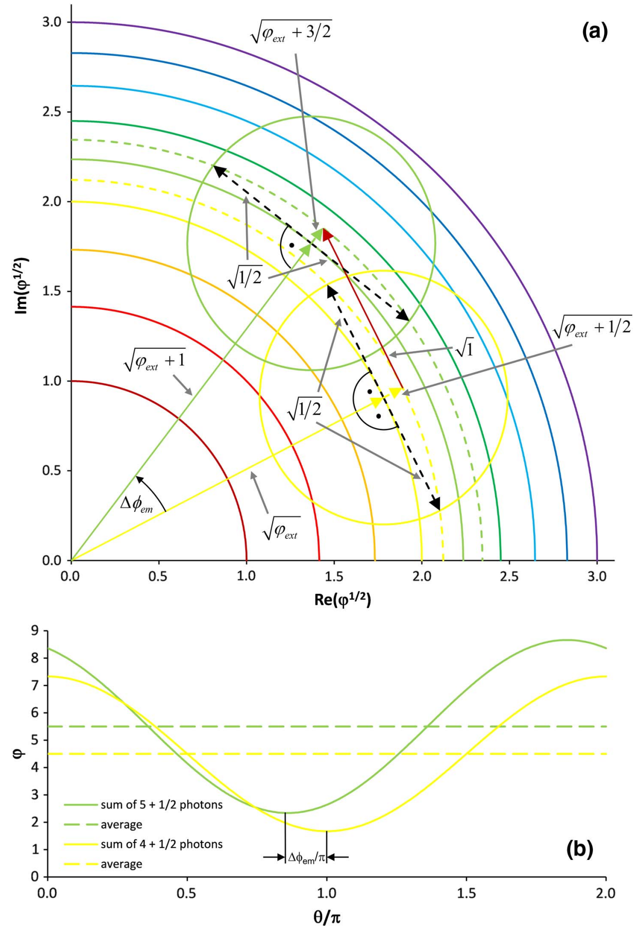

Fig. 8. Relation between vacuum fluctuations, violating the conservation of energy, and spontaneous emission, obeying the conservation of energy. (a) Quadrant of the amplitude-phase diagram illustrating the process of (stimulated and spontaneous) emission in the presence of vacuum fluctuations. In the example, an existing field representing 4 photons (solid yellow line and arrow) plus the vacuum fluctuation, i.e., in average 4.5 photons (dashed yellow line and arrow), is increased by the emission of a photon to a field representing 5 photons (solid light-green line and arrow) plus the vacuum fluctuation, i.e., in average 5.5 photons (dashed light-green line and arrow). The dashed black arrows indicate the addition of half a vacuum photon to the field of real photons, which can occur under any phase difference θ with the existing field, resulting in a state on the (yellow or green) circle, but averaging out over many such extremely short-lived events to a state on the (yellow or green) dashed line. Although occurring under all phase differences θ, the dashed black arrows are shown only for the two specific cases of $\pm 90°$ phase difference, where energy conservation happens not to be violated, equivalently to the intensity resulting from averaging over all phase angles. The dark-red arrow represents the field of the one emitted photon. The phase shift $\Delta\phi_{\text{em}}$ is calculated from Eq. (13). (b) Number φ of photons resulting from the interference according to Eq. (1) between ½ vacuum photon and 4 photons (yellow solid curve) or 5 photons (green solid curve) versus phase difference θ. Medium of 4.5 photons (yellow dashed line) or 5.5 photons (green dashed line) averaged over all θ. The deviations in amplitude from the average are consistent with the deviations predicted by the yellow and green rings in part (a).

$\varphi_{\text{ext}} + ½$ photons triggers atoms in their excited state to emit a photon, leading, according to Eq. (10), to a phase shift between triggering and transmitted field of

$$\tan(\Delta\phi_{\text{em}}) = \sqrt{1/(\varphi_{\text{ext}} + 1/2)}. \tag{13}$$

Spontaneous and stimulated emission are undistinguishable, because the total field interacts with each atom. Nevertheless, the increase in emission rate due to the additional half vacuum photon is quantified and measurable.

10. AMPLITUDE VERSUS PHASE FLUCTUATIONS, LASER LINEWIDTH, AND HENRY'S α-FACTOR

The assumptions that stimulated emission (absorption) occurs in (out of) phase with its driving field, whereas spontaneous emission occurs with an arbitrary phase difference, and their corresponding interpretation in the amplitude phase diagram of Fig. 1(a), have served as the foundation for the quantum-optical derivation of the laser linewidth in the 1960s. From the semi-classical point of view established above, one comes to the following judgement. The interpretation proposed by Lax [9], Haken [10], Henry [11], and others that spontaneous emission occurs with an arbitrary phase difference θ relative to an existing electromagnetic field, thereby introducing amplitude and phase fluctuations [projection onto the axes with $\theta = 0$–$180°$ and $\theta = \pm 90°$, respectively, in Fig. 1(a)], is not supported semi-classically, because it violates the law of energy conservation. Ironically, the $\theta = 90°$ phase difference of the spontaneously emitted photon that these authors considered to be a "pure phase fluctuation" manifests semi-classically exactly the amplitude addition by one photon [Fig. 5(a) or Fig. 8(a)] required to conserve the energy. From a semi-classical point of view, these authors have confused vacuum fluctuations with spontaneous emission.

If—in contrast to vacuum fluctuations—spontaneous emission induces neither phase nor amplitude fluctuations, these cannot explain any of the following phenomena. (i) The laser linewidth, i.e., the Schawlow–Townes linewidth [12] or any of its extended versions, is not a result of phase fluctuations. Without the existence of amplitude fluctuations, these cannot be damped out. Consequently, such a mechanism cannot explain (ii) the predicted [9,10,13–15] and experimentally observed [49,50] reduction of laser linewidth by a factor of 2 around threshold, nor will they (iii) induce the refractive-index changes and resulting amplitude–phase coupling and linewidth broadening originally proposed by Lax [9] and later quantified by Henry via his α-factor [11]. This finding does not exclude, however, that technical or other amplitude "fluctuations" occur, with all the described consequences.

11. CONCLUSIONS

Whatever aspect we have discussed semi-classically, (i) Maxwell's equations and the resulting interference term in the superposition of electromagnetic waves, (ii) energy conservation in a Fabry–Perot resonator, (iii) the Lorentz oscillator model, (iv) the Kramers–Kronig relations applied to the complex susceptibility, (v) the amplitude-phase diagram, or (vi) simply adding up sine waves in a quantized electric-wave picture, we have arrived at the conclusion that stimulated and spontaneous emission both occur under a 90° phase difference with the incident field. This semi-classical result also holds true as a standard perturbation-theoretical approach to absorption and stimulated emission in quantum optics [5–8,28] and, importantly, is supported by recent experimental evidence [38].

Naturally, the following questions arise: Can we find this 90° phase difference when applying the creation operator in quantum optics? If not, can we introduce it to the quantum-optical description, do we even have to rethink the quantum-optical description? Or is there a possibility that the semi-classical limit of quantum mechanics produces a 90° phase difference, whereas the more we enter the quantized world, the angle changes to 0°? It is hard to imagine a physical mechanism that allows us to derive a positive answer to the latter question.

The current explanation of the laser linewidth, as well as its narrowing and broadening, is in fundamental contradiction with the obtained and experimentally supported semi-classical picture. Recently, we have made significant progress toward establishing an explanation of the laser linewidth that coincides with the semi-classical picture of stimulated and spontaneous emission [51,52].

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REFERENCES

1. A. Einstein, "Zur Quantentheorie der Strahlung," Phys. Z. **18**, 121–128 (1917).
2. M. Planck, "Ueber das Gesetz der Energieverteilung im Normalspectrum," Ann. Phys. **309**, 553–563 (1901).
3. H. Kopfermann and R. Ladenburg, "Untersuchungen über die anomale Dispersion angeregter Gase. II. Teil. Anomale Dispersion in angeregtem Neon (Einfluß von Strom und Druck, Bildung und Vernichtung angeregter Atome)," Z. Phys. **48**, 26–50 (1928).
4. H. A. Lorentz, "Über die Beziehung zwischen der Fortpflanzungsgeschwindigkeit des Lichtes und der Körperdichte," Ann. Phys. **245**, 641–665 (1880).
5. M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., "Dipole oscillators," in *Laser Physics*, 6th ed. (Westview, 1993), Ch. 3.2, pp. 34–42.
6. P. W. Milonni, "Semiclassical and quantum-electrodynamical approaches in nonrelativistic radiation theory," Phys. Rep. **25**, 1–81 (1976).
7. M. Cray, M. L. Shih, and P. W. Milonni, "Stimulated emission, absorption, and interference," Am. J. Phys. **50**, 1016–1021 (1982).
8. P. W. Milonni and J. H. Eberly, "The electron oscillator model," in *Lasers* (Wiley, 1988), Ch. 2.2, pp. 27–33.
9. M. Lax, "Classical noise. V. Noise in self-sustained oscillators," Phys. Rev. **160**, 290–307 (1967).
10. H. Haken, *Laser Theory*, Vol. **XXV/2c** of Encyclopedia of Physics (Springer, 1970).
11. C. H. Henry, "Theory of the linewidth of semiconductor lasers," IEEE J. Quantum Electron. **18**, 259–264 (1982).
12. A. L. Schawlow and C. H. Townes, "Infrared and optical masers," Phys. Rev. **112**, 1940–1949 (1958).
13. P. Grivet and A. Blaquière, "Masers and classical oscillators," in *Symposium on Optical Masers*, J. Fox, ed. (Polytechnique Institute, 1963), pp. 69–93.
14. H. Risken, "Correlation function of the amplitude and of the intensity fluctuations for a laser model near threshold," Z. Phys. **191**, 302–312 (1966).
15. R. D. Hempstead and M. Lax, "Classical noise. VI. Noise in self-sustained oscillators near threshold," Phys. Rev. **161**, 350–366 (1967).
16. R. J. Glauber, "Coherent and incoherent states of the radiation field," Phys. Rev. **131**, 2766–2788 (1963).
17. R. de L. Kronig, "On the theory of the dispersion of x-rays," J. Opt. Soc. Am. **12**, 547–557 (1926).
18. H. A. Kramers, "La diffusion de la lumière par les atoms," in *Atti del Congresso Internazionale del Fisica (Transactions of Volta Centenary Congress) Como* (1927), Vol. **2**, pp. 545–557.

19. H. S. Sommers, “Spontaneous power and the coherent state of injection lasers,” J. Appl. Phys. **45**, 1787–1793 (1974).
20. H. S. Sommers, “Threshold and oscillation of injection lasers: a critical review of laser theory,” Solid State Electron. **25**, 25–44 (1982).
21. A. E. Siegman, “Oscillation dynamics and oscillation threshold,” in *Lasers* (University Science Books, 1986), Ch. 13, pp. 510–524.
22. G. Björk and Y. Yamamoto, “Analysis of semiconductor microcavity lasers using rate equations,” IEEE J. Quantum Electron. **27**, 2386–2396 (1991).
23. W. Heisenberg, “Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik,” Z. Phys. **43**, 172–198 (1927).
24. H. P. Robertson, “The uncertainty principle,” Phys. Rev. **34**, 163–164 (1929).
25. J. C. Maxwell, “VIII. A dynamical theory of the electromagnetic field,” Philos. Trans. R. Soc. London **155**, 459–512 (1865).
26. P. A. M. Dirac, “The quantum theory of the emission and absorption of radiation,” Proc. R. Soc. A **114**, 243–265 (1927).
27. L. I. Schiff, “Interaction between charged particles and the electromagnetic field,” in *Quantum Mechanics*, 3rd ed. (McGraw-Hill, 1968), Sec. 57, pp. 531–533.
28. L. I. Schiff, “Absorption and induced emission,” in *Quantum Mechanics*, 3rd ed. (McGraw-Hill, 1968), Sec. 44, p. 398–406.
29. E. Fermi, *Nuclear Physics* (The University of Chicago, 1950), formula VIII.2.
30. R. P. Feynman, R. B. Leighton, and M. Sands, “The origin of the refractive index,” in *The Feynman Lectures on Physics* (Addison-Wesley, 1963), Vol. **I**, Ch. 31, pp. 31–34.
31. V. F. Weisskopf, “How light interacts with matter,” Sci. Am. **219**, 60–71 (1968).
32. M. Motsch, M. Zeppenfeld, P. W. H. Pinkse, and G. Rempe, “Cavity-enhanced Rayleigh scattering,” New J. Phys. **12**, 063022 (2010).
33. N. Ismail, C. C. Kores, D. Geskus, and M. Pollnau, “Fabry–Pérot resonator: spectral line shapes, generic and related Airy distributions, linewidths, finesses, and performance at low or frequency-dependent reflectivity,” Opt. Express **24**, 16366–16389 (2016).
34. M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., “Coherent states,” in *Laser Physics*, 6th ed. (Westview, 1993), Ch. XV, pp. 242–255.
35. R. H. Dieke, “Coherence in spontaneous radiation processes,” Phys. Rev. **93**, 99–110 (1954).
36. M. Gross and S. Haroche, “Superradiance: an essay on the theory of collective spontaneous emission,” Phys. Rep. **93**, 301–396 (1982).
37. M. Scheibner, T. Schmidt, L. Worschech, A. Forchel, G. Bacher, T. Passow, and D. Hommel, “Superradiance of quantum dots,” Nat. Phys. **3**, 106–110 (2007).
38. A. Sommer, E. M. Bothschafter, S. A. Sato, C. Jakubeit, T. Latka, O. Razskazovskaya, H. Fattahi, M. Jobst, W. Schweinberger, V. Shirvanyan, V. S. Yakovlev, R. Kienberger, K. Yabana, N. Karpowicz, M. Schultze, and F. Krausz, “Attosecond nonlinear polarization and light-matter energy transfer in solids,” Nature **534**, 86–90 (2016).
39. A. Kastler, “Review: masers and lasers,” Eur. J. Phys. **7**, 69–76 (1986).
40. G. Bruhat and A. Kastler, “Interférences de la lumière,” in *Optique*, 6th ed. (Masson, 1965), pp. 60–61, 95–96, and 175.
41. T. V. Raziman and O. J. F. Martin, “Does the real part contain all the physical information?” J. Opt. **18**, 095002 (2016).
42. J. A. Armstrong, N. Bloembergen, J. Ducuing, and P. S. Pershan, “Interactions between light waves in a nonlinear dielectric,” Phys. Rev. **127**, 1918–1939 (1962).
43. E. Rosencher and C. Fabre, “Oscillation characteristics of continuous-wave optical parametric oscillators: beyond the mean-field approximation,” J. Opt. Soc. Am. B **19**, 1107–1116 (2002).
44. A. E. Siegman, “Excess spontaneous emission in non-Hermitian optical systems. I. Laser amplifiers,” Phys. Rev. A **39**, 1253–1263 (1989).
45. A. E. Siegman, “Excess spontaneous emission in non-Hermitian optical systems. II. Laser oscillators,” Phys. Rev. A **39**, 1264–1268 (1989).
46. W. A. Hamel and J. P. Woerdman, “Nonorthogonality of the longitudinal eigenmodes of a laser,” Phys. Rev. A **40**, 2785–2787 (1989).
47. A. M. van der Lee, N. J. van Druten, A. L. Mieremet, M. A. van Eijkelenborg, Å. M. Lindberg, M. P. van Exter, and J. P. Woerdman, “Excess quantum noise due to nonorthogonal polarization modes,” Phys. Rev. Lett. **79**, 4357–4360 (1997).
48. M. Eichhorn and M. Pollnau, “Spectroscopic foundations of lasers: spontaneous emission into a resonator mode,” IEEE J. Sel. Top. Quantum Electron. **21**, 9000216 (2015).
49. H. Gerhardt, H. Welling, and A. Güttner, “Measurements of the laser linewidth due to quantum phase and quantum amplitude noise above and below threshold. I,” Z. Phys. **253**, 113–126 (1972).
50. H. Gerhardt, H. Welling, and A. Güttner, “Observation of quantum-phase and quantum-amplitude noise for a laser below and above threshold,” Phys. Lett. A **40**, 191–193 (1972).
51. M. Eichhorn and M. Pollnau are preparing a manuscript to be called “Laser linewidth, spectral coherence, and the laser eigenvalue.”
52. M. Pollnau is preparing a manuscript to be called “Reconciling Einstein, Planck, and Heisenberg: the factor of two in vacuum energy and laser linewidth.”